Design and selection of springs

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Outline

- **Functions and types**
- **Helical coil springs (compression and extension or tension)**
  – Consideration for selection of springs
  – Selection of compression and extension springs using manufacturer's catalogue
- **Other types of springs**
  – Belleville washers (disc springs)
  – Design parameters and method of stacking
  – Selection of disc springs using manufacturer's catalogue

Functions of springs

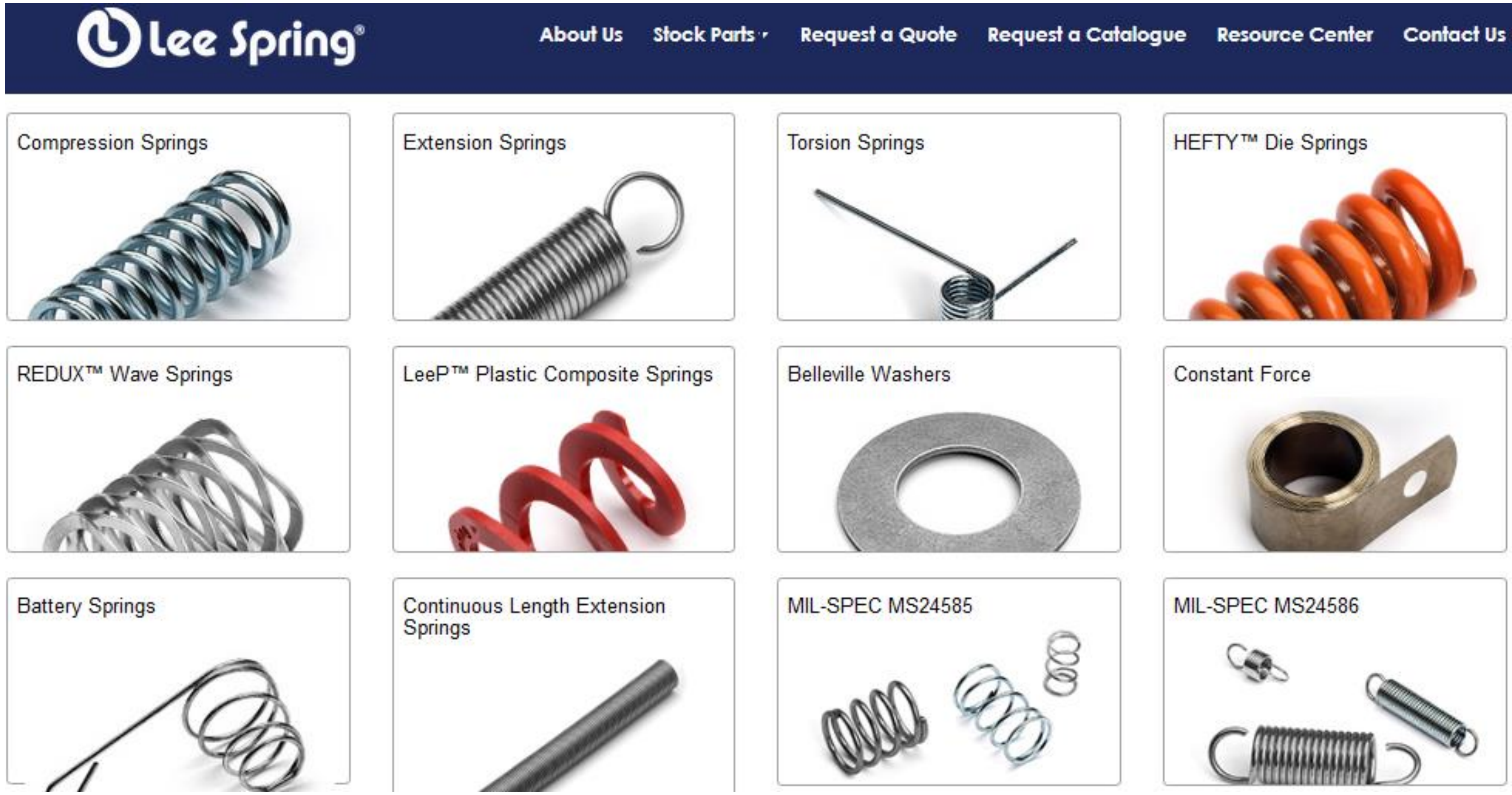
- **Springs** are flexible machine components that deflects under the action of load and returns to its original shape when load is removed
- **Springs** are often used to
  - absorb and store energy, e.g. vehicle suspension, railway buffers to control energy, buffer springs in elevators and vibration mounts for machinery
  - Measure forces, e.g. spring balances, gages
  - Apply force and control motion

Types of springs

- Helical coil springs
  - Helical compression, extension and torsion springs
- Belleville washers
- Torsion bar springs
- Leaf springs
- Volute springs

leaf Spring

Volute Spring

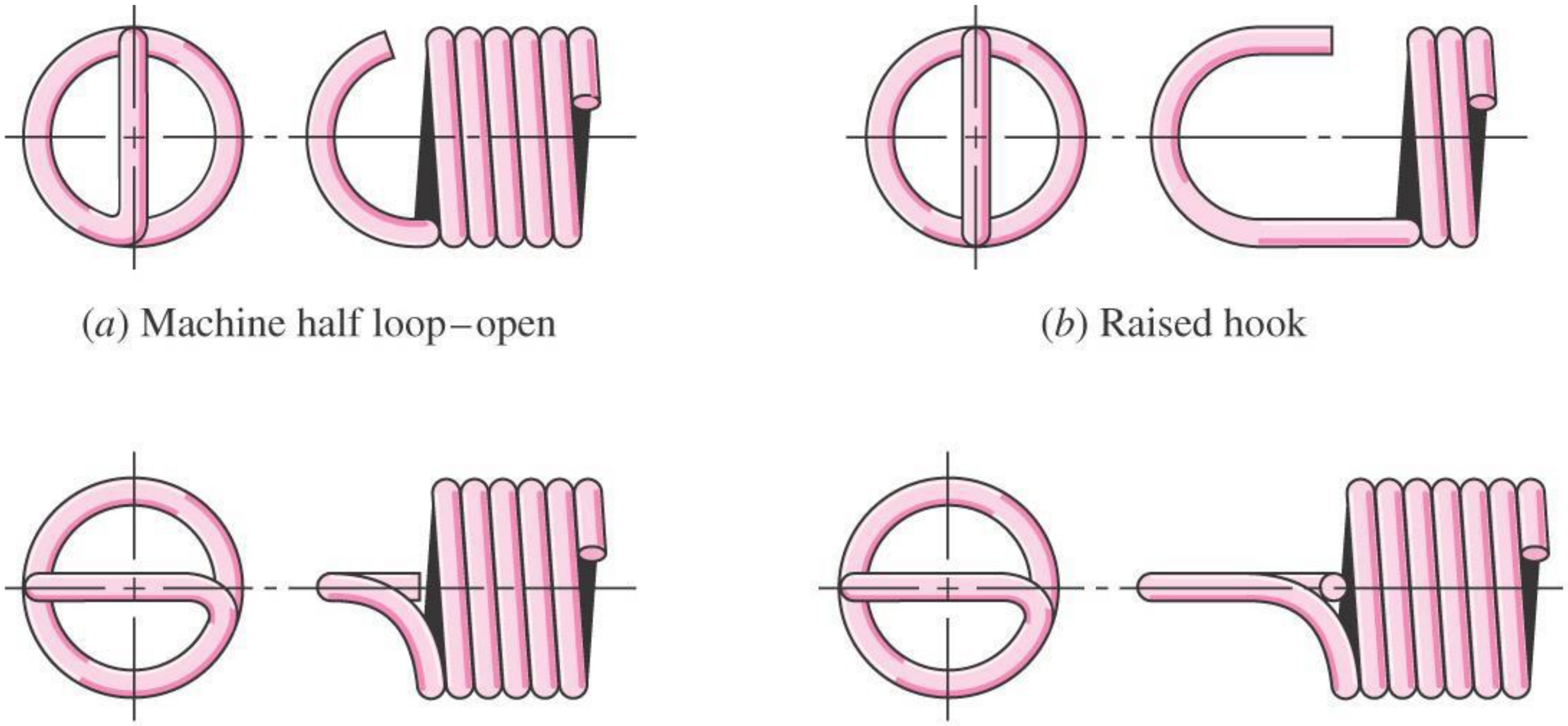


Lee Spring online catalogue (https://www.leespring.co.uk/)

https://advanex.co.uk/productsandservices/

Helical coil extension (tension) springs

- **Helical coil extension or tension springs** are commonly used to store energy or allow tension force to be applied
- To apply tensile loads, **hooks** are needed at the ends of the springs.



(*a*) Machine half loop–open

(*b*) Raised hook

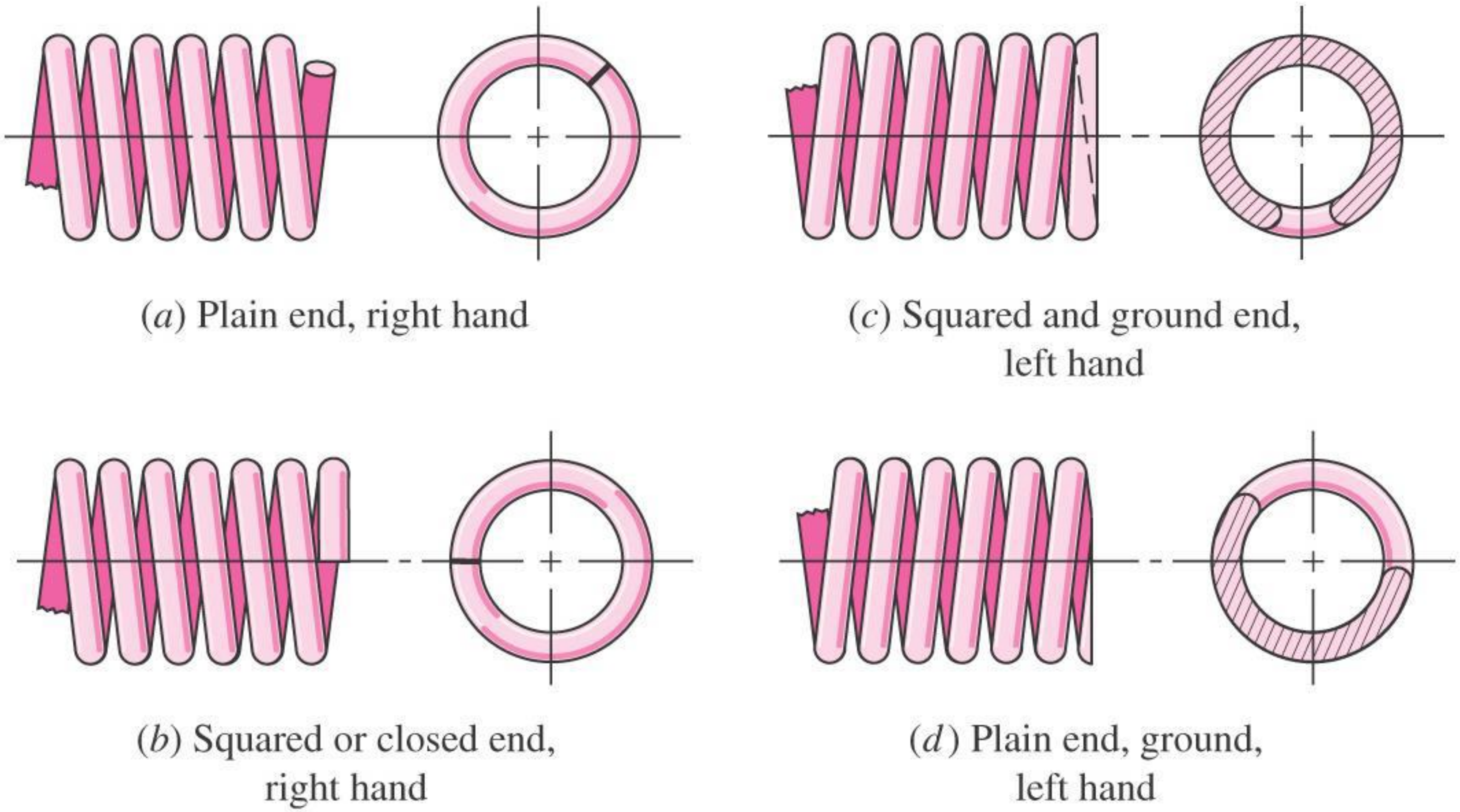
(*c*) Short twisted loop

(*d*) Full twisted loop

End forms of extension coil springs

Helical coil compression springs

- **Helical coil compression springs** are commonly used to store energy or provide resistant axial force under compression.



End forms of compression coil springs

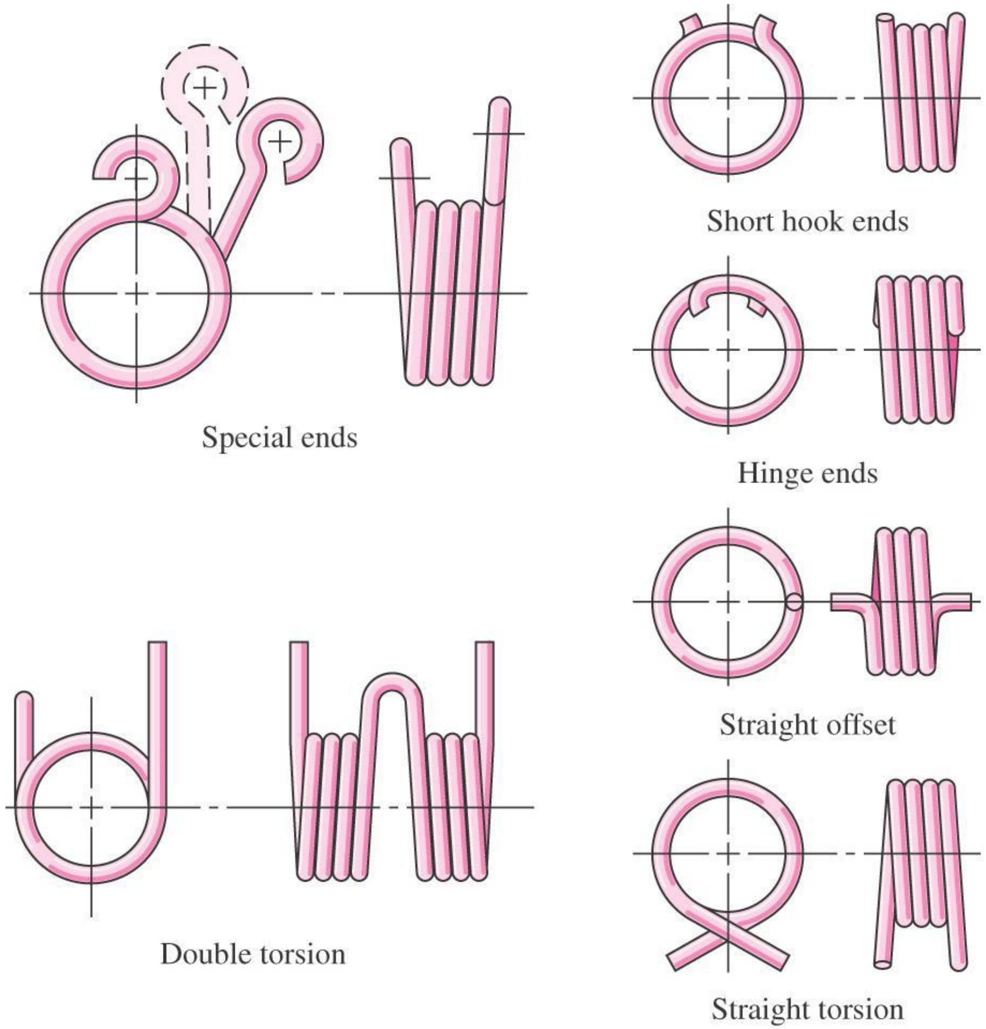
http://springipedia.com/

Helical coil torsion springs

- **Torsion springs** are used to **apply a torque or store rotational energy**

- When the **ends of torsion springs** are twisted around the centre of the spring, the torsion spring tries to push them back to their original position.

- Like tension springs, **torsion springs** are usually close-wound. There are **single- and double-bodied types** with **short hook, hinged straight offset, straight torsion, and special ends**.



Types and end forms of torsion springs

Spring rate (extension)

➢ The **Spring rate equation** for extension springs

$$\boldsymbol{k} = \frac{\boldsymbol{F}}{\boldsymbol{\delta}}\ (N/mm)$$

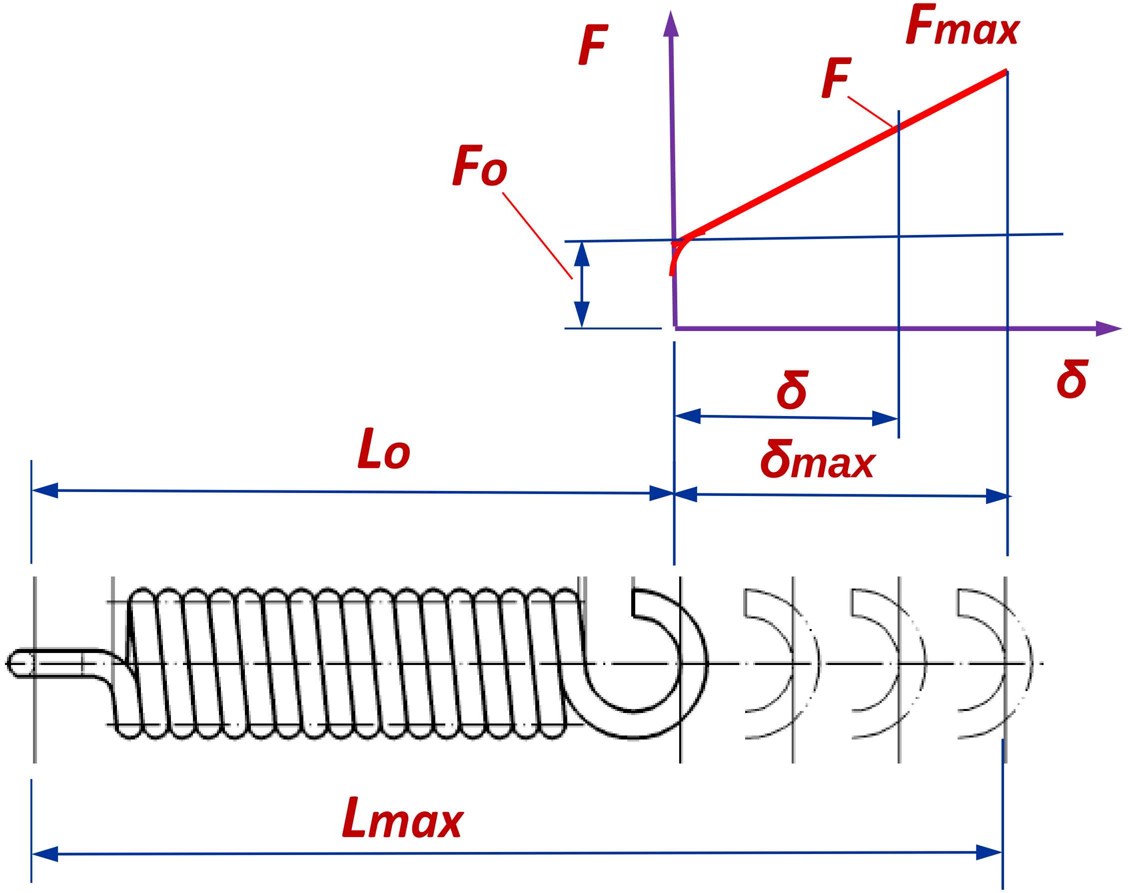
$$\boldsymbol{k} = \frac{\boldsymbol{F_{max}} - \boldsymbol{F_o}}{\boldsymbol{L_{max}} - \boldsymbol{L_o}}\ (N/mm)$$

or

$$\boldsymbol{k} = \frac{\boldsymbol{F} - \boldsymbol{F_o}}{\boldsymbol{\delta}}\ (N/mm)$$

$$\delta = L - L_o$$

Where, ***Lo*** is free length, ***Lmax*** is the max extended length, ***Fo*** is preload and ***Fmax*** is the max force at the mas extended length



Various lengths of a extension spring

Spring rate (compression)

- **Spring rate** is slope of the force (***F***) –deflection (***δ***) curve of a spring under either compression or tension

$$k = \frac{F}{\delta}\ (N/mm)$$

- With given preload (***Fa***) and installed length (***La***), and known operation load (***Fo***) and length (***Lo***),

$$k = \frac{F_o - F_a}{L_a - L_o}\ (N/mm)$$

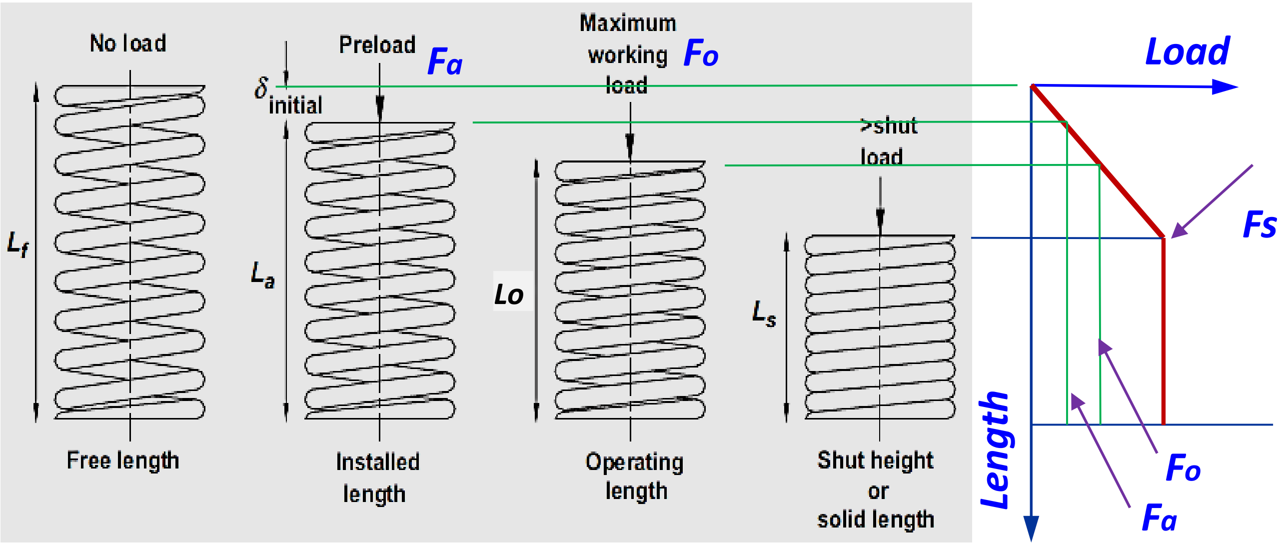
- The free length (***Lf***) & length (***Ls***) are

$$L_f = L_a + \frac{F_a}{k}\ (mm)$$

$$L_s = dN(mm)$$

d is wire diameter

N is total number of coils of the spring



Lengths of compression spring

Springs in series and parallel

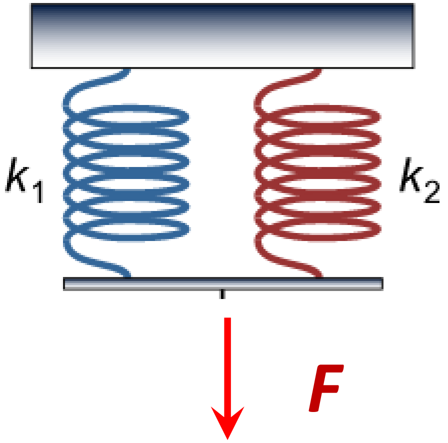
- Springs may be arranged in different ways of combination

- **Springs in parallel**

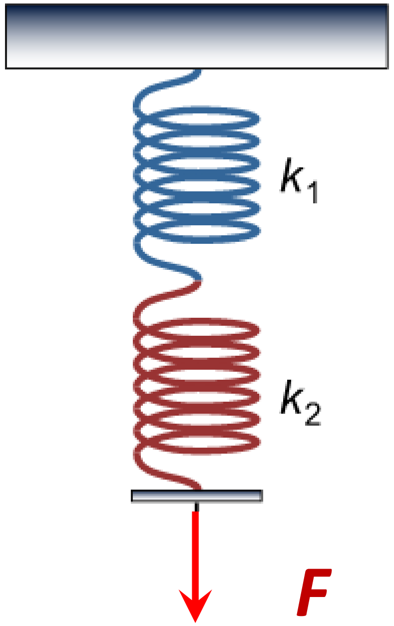
$$k_{total} = \sum_{i=1}^{n} k_i = k_1 + k_2 + \cdots$$

- **Springs in series**

$$\frac{1}{k_{total}} = \sum_{i=1}^{n} \frac{1}{k_i} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$



Springs in parallel



Springs in series

Springs in parallel:

$$F = F_1 + F_2$$
$$\delta = \delta_1 = \delta_2$$

Springs in series:

$$F = F_1 = F_2$$
$$\delta = \delta_1 + \delta_2$$

$$F = k\delta, \quad F_1 = k_1\delta_1, F_2 = k_2\delta_2$$

Springs in parallel:

$$k = k_1 + k_2$$

Springs in series:

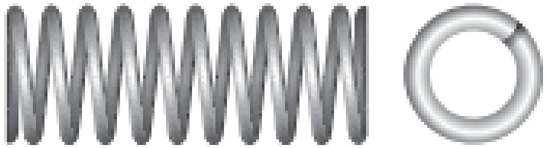
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

Design and selection of springs

- **General steps to select a suitable spring**
  - Understand a suitable **type**, **load range**, required **free, compressed or extended length** and **spring rate** to meet requirement in **given space;**
  - Know operational conditions for sufficient life without buckling or permanent (creep) deformation;
  - **Choose suitable material** (music wire, stainless steel, bronze, etc) for sufficient energy store and force capabilities;
  - Make sure **natural frequencies** of vibration much higher than the frequency of spring motion that they control;
  - Meet **other design requirements**, e.g. cost, easy access for assembly;
  - **Select a suitable spring from a Spring manufacturer's catalogue, e.g. *https://www.leespring.co.uk/***

Selection of springs

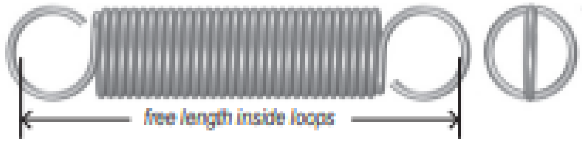
➢ Selection of springs from **suppliers** and **manufacturers** knowing **required force, spring rate, deflection range, general operational conditions, etc**.



COMPRESSION SPRINGS

● End Coils Closed and Ground Square ● Music Wire (Plated) or Stainless Steel (Passivated)

LEE STOCK NUMBER	OUTSIDE DIAMETER		TO WORK IN HOLE DIA. MIN		NOMINAL WIRE DIAMETER		TO WORK OVER ROD DIA. MAX		APPROXIMATE LOAD AT SOLID HEIGHT		NOMINAL FREE LENGTH		SPRING RATE		APPROXIMATE SOLID HEIGHT		PRICE GROUP		
																	Music Wire	302 Stainless	316 Stainless
	MM	IN	MM	IN	MM	IN	MM	IN	N	LB	MM	IN	N/MM	LB/IN	MM	IN	M	S	S316
LCM160H 06	15.00	0.591	16.00	0.630	1.60	0.063	11.00	0.433	101.99	22.93	35.00	1.378	4.62	26.40	12.95	0.510	K	M	SPECIAL
LCM160H 07											40.00	1.575	3.99	22.80	14.48	0.570	K	M	SPECIAL
LCM160H 08																			



EXTENSION SPRINGS

● Loops at Random Position, except for † springs ● Music Wire (Plated), or Stainless Steel (Passivated)

LEE STOCK NUMBER	OUTSIDE DIAMETER		NOMINAL WIRE DIAMETER		MAXIMUM LOAD		INITIAL TENSION		NOMINAL FREE LENGTH		SPRING RATE		MAXIMUM EXTENDED LENGTH		PRICE GROUP	
															Music Wire	302 Stainless
	MM	IN	MM	IN	N	LB	N	LB	MM	IN	N/MM	LB/IN	MM	IN	M	S
LE 095J 03	25.40	1.000	2.41	0.095	133.45	30.00	12.01	2.70	88.90	3.500	1.524	8.70	168.66	6.640	Z	BF
LE 095J 04									101.60	4.000	1.208	6.90	202.18	7.960	BA	BG
LE 095J 05									114.30	4.500	0.981	5.60	238.25	9.380	BB	BH

Lee Spring® catalogue at
https://www.leespring.co.uk/

A worked example: Selection of compression spring for a spring supplier (LeeSpring)

➢ Select a compression spring required to exert a force of **35 N** when compressed to a length of **60 mm**. At a length of **48 mm**, the force must be **50 N**. The spring is to be installed in a **hole with 24 mm diameter**.

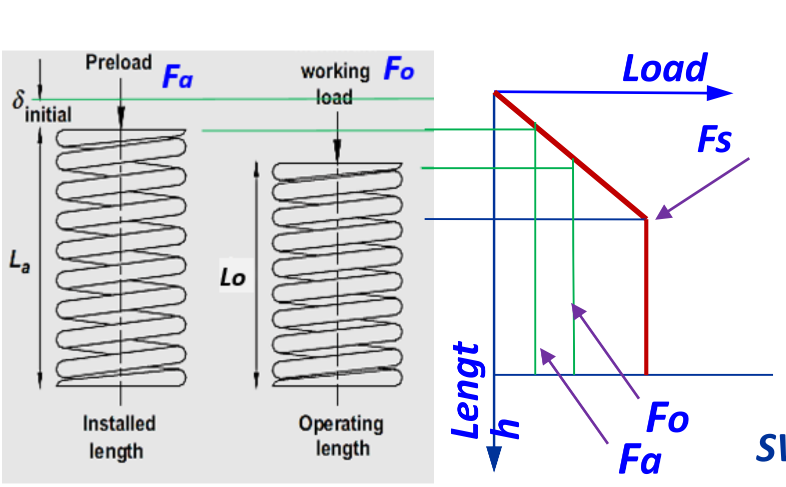
From the above, we know

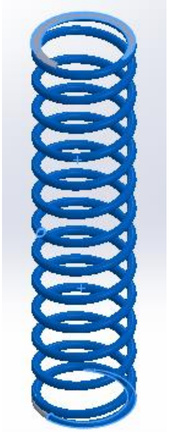
$$F_a = 35\ N,\ L_a = 60\ mm\ \&\ F_o = 50\ N,\ L_o = 48\ mm$$

From slide 9,

$$k = \frac{F_o - F_a}{L_a - L_o} = \frac{50 - 35}{60 - 48} = 1.25\ (N/mm)$$

$$L_f = 60 + \frac{35}{1.25} = 88\ (mm)$$

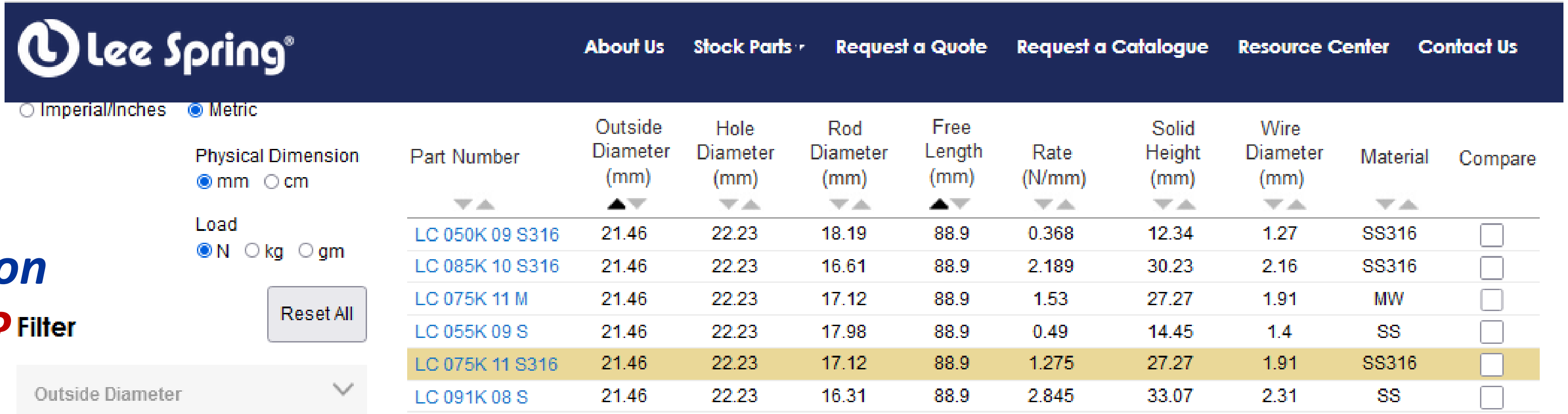




SW model download from LeeSpring®

Check Lee Spring® catalogue at https://www.leespring.co.uk/

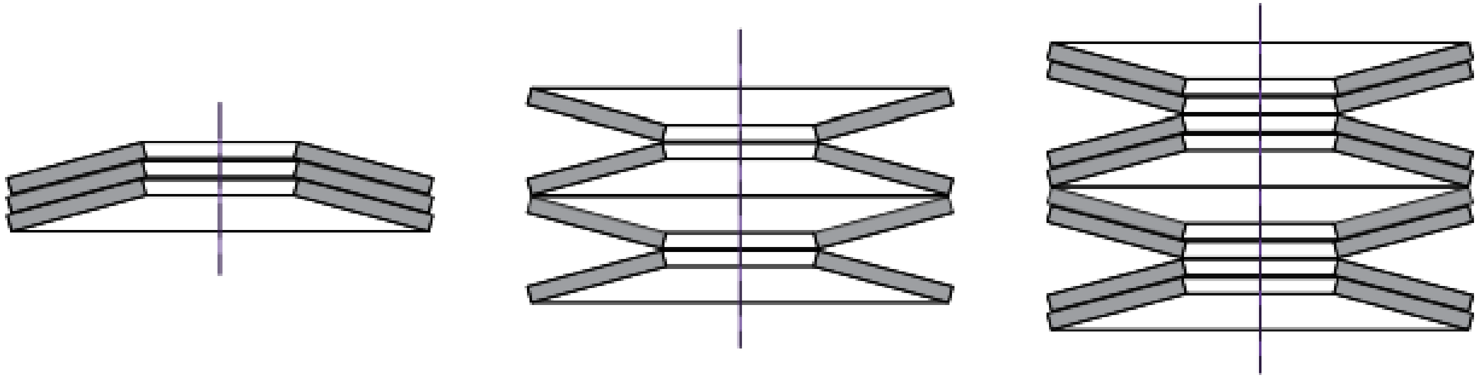
Can you select a compression spring for a mountain bike?



Part Number	Outside Diameter (mm)	Hole Diameter (mm)	Rod Diameter (mm)	Free Length (mm)	Rate (N/mm)	Solid Height (mm)	Wire Diameter (mm)	Material	Compare
LC 050K 09 S316	21.46	22.23	18.19	88.9	0.368	12.34	1.27	SS316	
LC 085K 10 S316	21.46	22.23	16.61	88.9	2.189	30.23	2.16	SS316	
LC 075K 11 M	21.46	22.23	17.12	88.9	1.53	27.27	1.91	MW	
LC 055K 09 S	21.46	22.23	17.98	88.9	0.49	14.45	1.4	SS	
LC 075K 11 S316	21.46	22.23	17.12	88.9	1.275	27.27	1.91	SS316	
LC 091K 08 S	21.46	22.23	16.31	88.9	2.845	33.07	2.31	SS	

Belleville washers (Disc springs)

- Belleville washers are specific springs of conical-shaped discs
  - **High load capacity** with small deflection in confined space
  - Inherent dampening with parallel stacking
  - Flexibility in **stack arrangement** to meet different operational requirem



IN PARALLEL

Deflection: Same as single Disc

Force: Single Disc multiplied by the number of Discs

IN SERIES

Deflection: Single Disc multiplied by the number of Discs

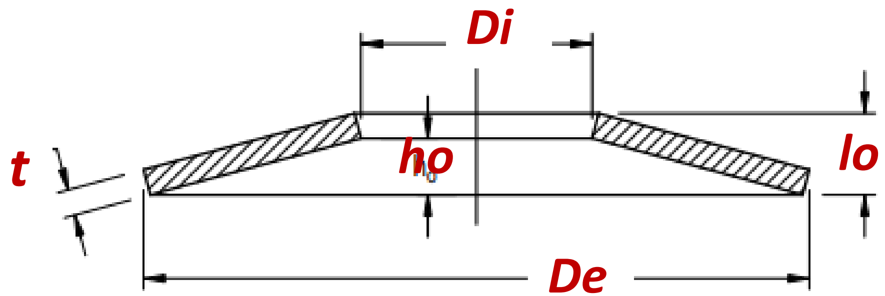
Force: Same as single Disc

IN COMBINATION

Deflection: Single Disc multiplied by the number of Discs in series

Force: Single Disc multiplied by the number of parallel Discs in a set

Same concept as springs in parallel and series



D_e = External Diameter of Disc

D_i = Internal Diameter of Disc

l_o = Free Height of Disc

t = Material Thickness of Disc

h_o = Free Cone Height of Disc

Spirol disc springs design guide also available on **Moodle**

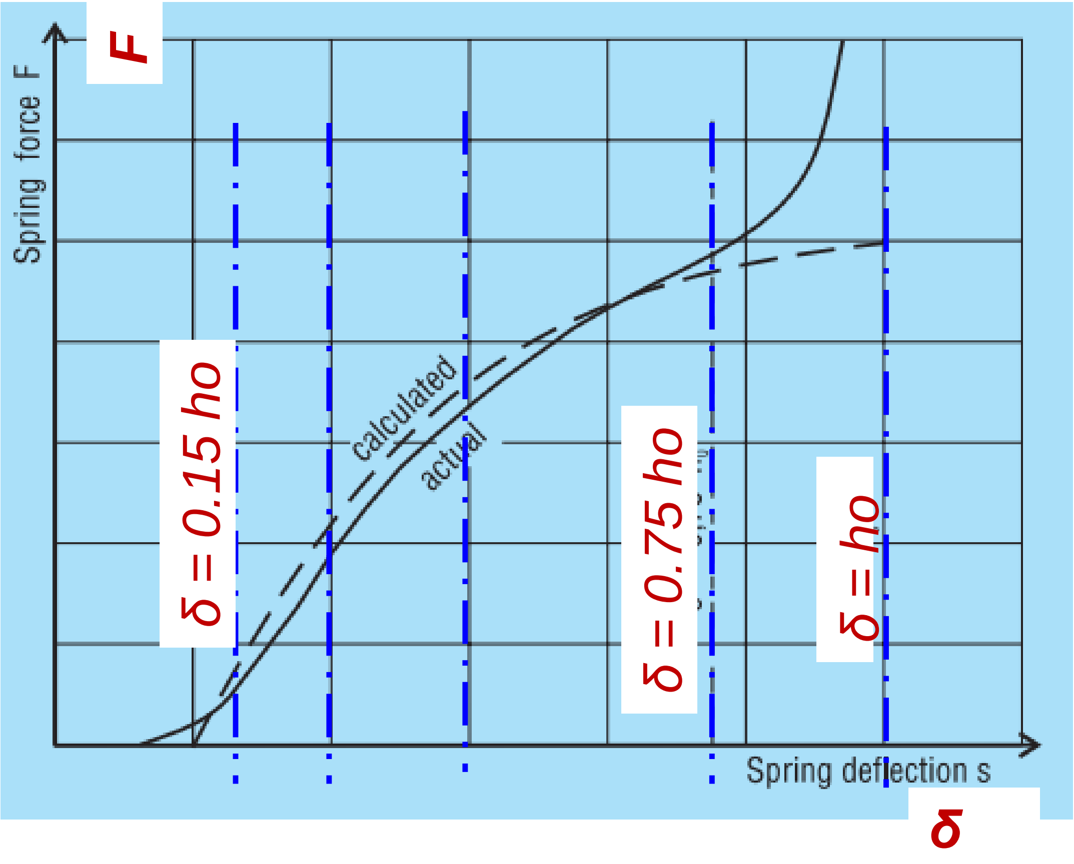
Force and deflection relationship

- Unlike compression and tension springs, Belleville washers have **non-linear force and deflection** relationship.

$$F = \frac{4E\delta}{K_1 D_e^2 (1 - \nu^2)} \left[(h - \delta)\left(h - \frac{\delta}{2}\right) t + t^3 \right]$$

Where, $K_1 = \frac{6}{\pi \ln R_d}\left[\frac{(R_d - 1)^2}{R_d^2}\right]$ $\quad R_d = \frac{D_e}{D_i}$

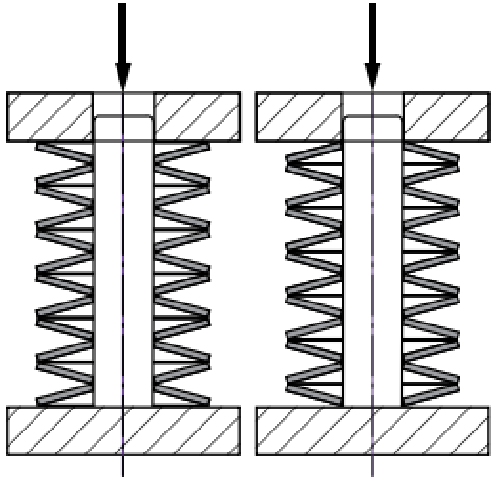
Force/deflection predictability is limited to **75%** of total deflection (***ho***). Therefore, ***0.15 ho & 0.75 ho are often used*** as the initial & max compressions in **Design & Selection of Belleville washers (Disc springs).**

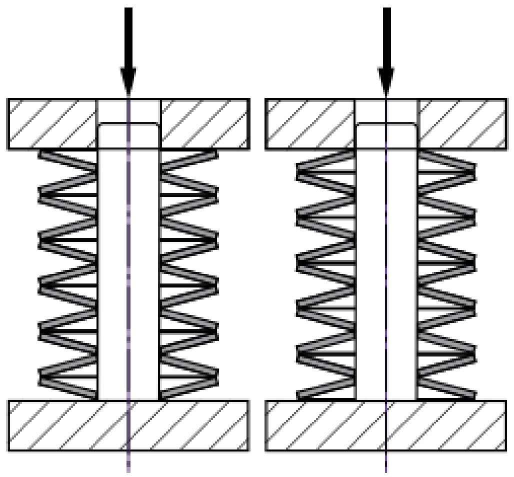


Belleville washer force-deflection curve

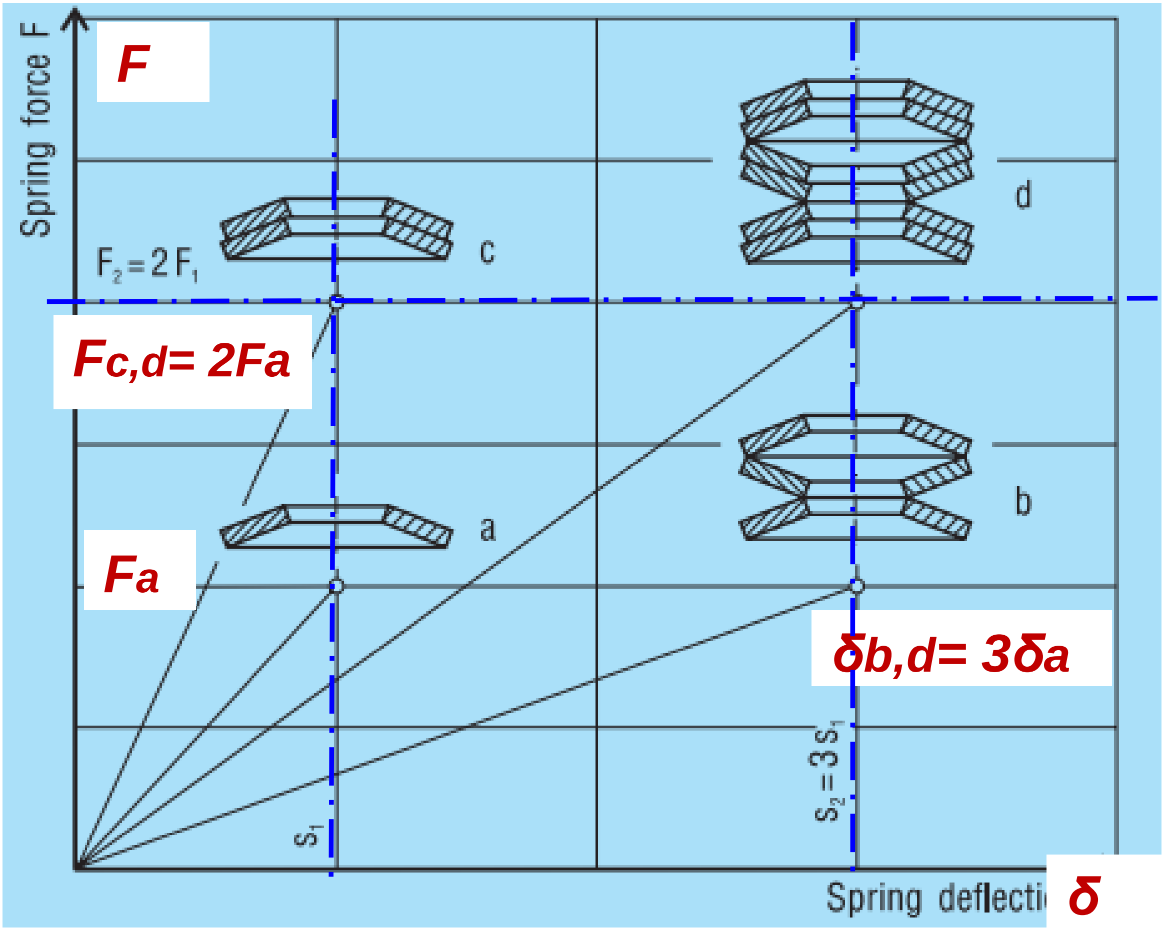
Stack combinations

- Different force and deflection relationships may be achieved by using different **stack combinations**





Use **outer edge to rest on both ends**. In the case of **odd numbers of stacks**, the end with **outer edge should be on the end of force/motion.**

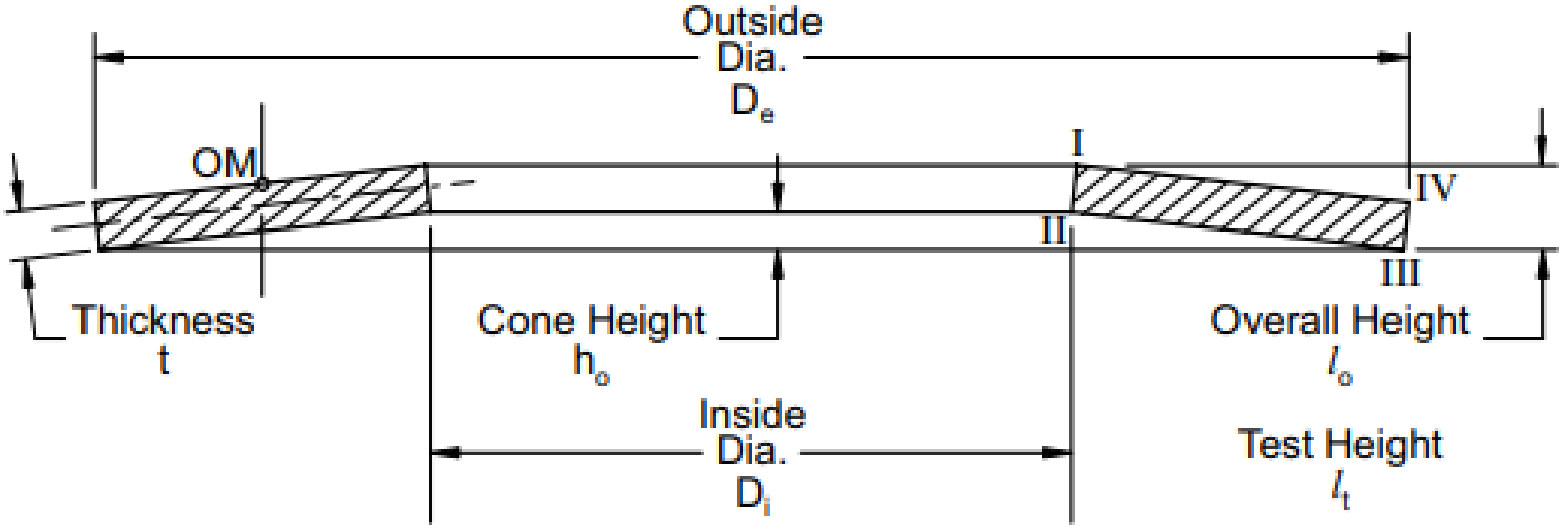


Stack combinations to give different Force and deflection characteristics

Selection of disc springs

- Similar to coil springs, suitable Belleville washers (disc springs) may be chosen from suppliers and manufacturers
  - **Define outer/inner diameters, force & deflection range**
  - **Select a single disc spring with data at *0.15ho* and *0.75ho***
  - **Work out a suitable stack combination to meet the requirements of total force (*F*), deflection (*δ* or *s*) and outer/inner diameters (*De*/*Di*).**

Spirol Disc Springs guide to DIN EN 16983 also available on Moodle

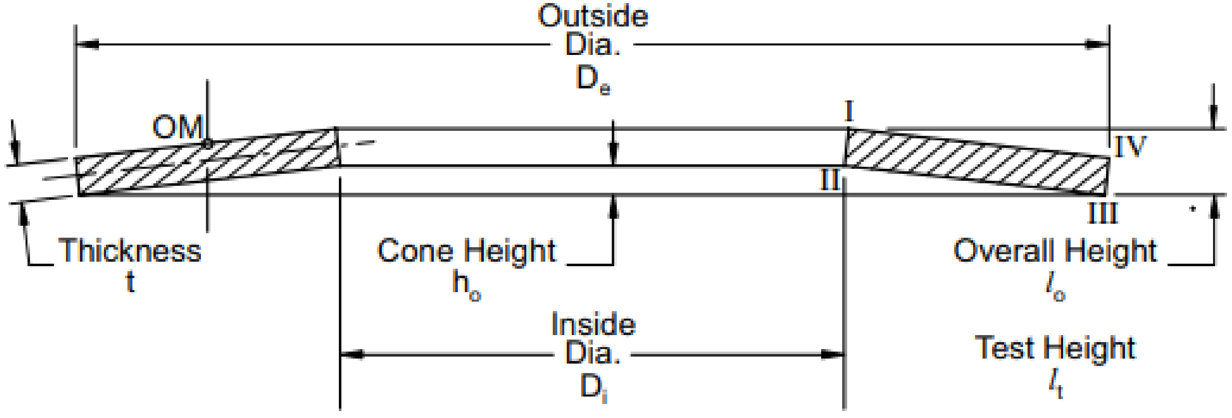


In addition to the choice of different sizes, disc springs

- **in parallel** for larger force
- **in series** for larger deflection

Selection of Disc springs

➤ Spirol Disc Spring (Belleville washer) catalogue



Deflection s in mm
Force F in N
Stress σ in MPa
Values calculated in accordance with DIN EN 16984 (formerly DIN 2092)

STANDARD MATERIALS		
B	"t" less than 1.25mm High Carbon Steel	HV 425 - 510 HRC 43 - 50
W	"t" 1.25mm and thicker Alloy Steel	HV 412 - 544 HRC 42 - 52
STANDARD FINISH		
R	Phosphate coated, oiled	

DIN Series	Dimensions						Design Force, Deflection and Stresses Based on E = 206 kMPa and μ = 0.3																											
							Preload, $s = 0.15 h_o$					$s = 0.25 h_o$					$s = 0.5 h_o$					$s = 0.75 h_o$					$s = h_o$		
	D_e	D_i	t	l_o	h_o	h_o/t	s	l_t	F	σ_{II}	σ_{III}	s	l_t	F	σ_{II}	σ_{III}	s	l_t	F	σ_{II}	σ_{III}	s	l_t	F	σ_{II}	σ_{III}	s	F	σ_{OM}
	8.0	3.2	0.20	0.40	0.20	1.00	0.03	0.37	8	37	144	0.05	0.35	12	97	276	0.10	0.30	20	211	433	0.15	0.25	26	409	600	0.20	30	-710
	8.0	3.2	0.30	0.55	0.25	0.83	0.04	0.51	29	113	247	0.06	0.49	46	207	401	0.13	0.43	79	511	750	0.19	0.36	104	912	1,046	0.25	126	-1,332
B	50.0	25.4	2.00	3.40	1.40	0.70	0.21	3.19	1,226	128	264	0.35	3.05	1,949	230	430	0.70	2.70	3,491	537	810	1.05	2.35	4,762	923	1,140	1.40	5,898	-1,408
	50.0	25.4	2.25	3.75	1.50	0.67	0.23	3.53	1,821	165	312	0.38	3.38	2,905	292	508	0.75	3.00	5,249	675	959	1.13	2.63	7,217	1,147	1,353	1.50	8,997	-1,697
	50.0	25.4	2.50	3.90	1.40	0.56	0.21	3.69	2,154	204	302	0.35	3.55	3,473	355	494	0.70	3.20	6,437	789	938	1.05	2.85	9,063	1,301	1,332	1.40	11,519	-1,760
A	50.0	25.4	3.00	4.10	1.10	0.37	0.17	3.94	2,594	249	249	0.28	3.83	4,255	424	409	0.55	3.55	8,214	897	787	0.83	3.27	11,976	1,418	1,135	1.10	15,640	-1,659

(https://www.spirol.com/library/main_catalogs/SPIROL-Disc-Springs-us.pdf), also available on Moodle

A worked example: Selection of disc spring

- Select **Spirol disc springs** required to exert an initial force of **200 N** at ***0.15ho*** and a maximum operational force of **700 N** under further **2.7 mm** deflection. The spring is to be installed in a **hole with 24 mm diameter**.

From the above, we know

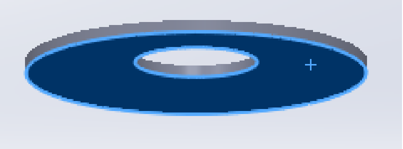
$$s1 = 0.15ho \rightarrow Fa1 = 200\ N\ \&\ s2 = 0.75ho \rightarrow Fa2 = 700\ N$$

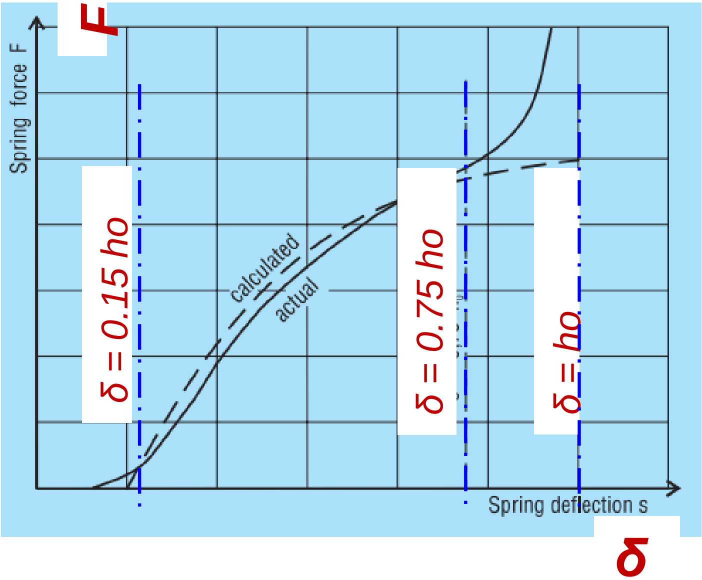
From Spirol disc spring table,

$$s1 = 0.15ho = 0.11\ mm \rightarrow Fa1 = 214\ N$$

$$s2 = 0.75ho = 0.56\ mm \rightarrow Fa2 = 719\ N$$

$\Delta s = S2 - S1 = 0.6ho$ =0.45 (mm) → N=S/ Δs =2.7/0.45 = 6 <u>(springs)</u>





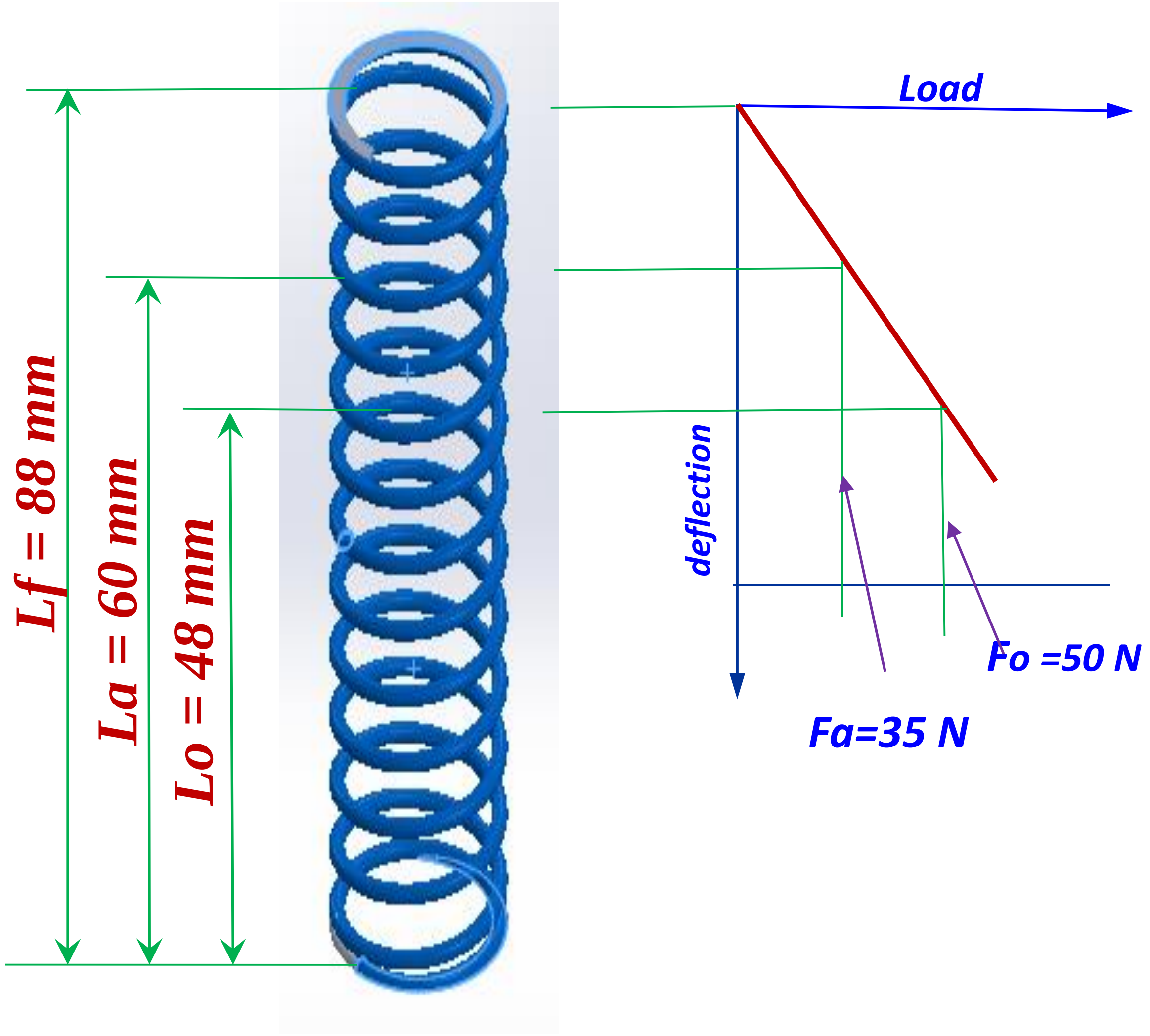
SW model download from LeeSpring®

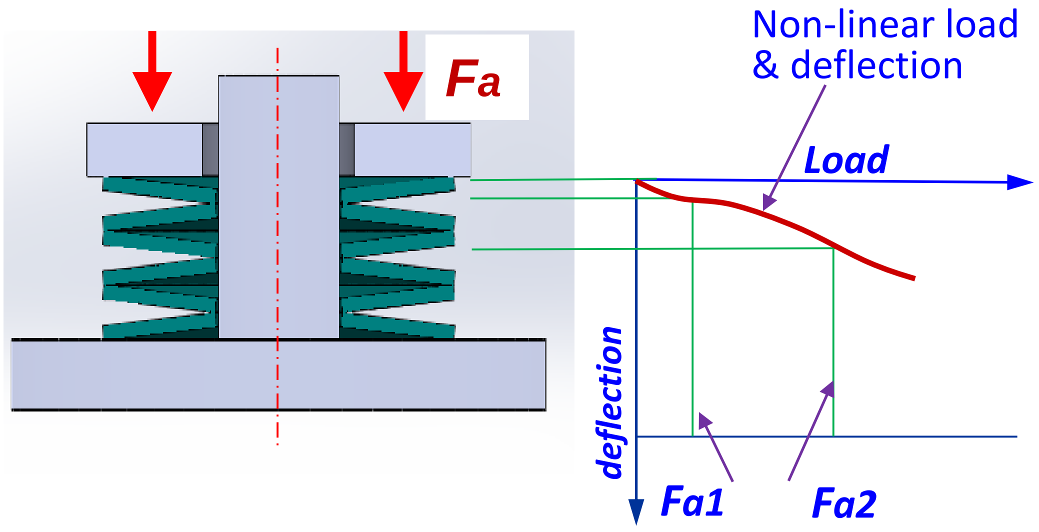
DISC SPRINGS TO DIN EN 16983 (formerly DIN 2093)

DIN Series	Dimensions						Design Force, Deflection and Stresses Based on E = 206 kMPa and μ = 0.3																							
							Preload, $s = 0.15 h_o$					$s = 0.25 h_o$					$s = 0.5 h_o$					$s = 0.75 h_o$					$s = h_o$		
	D_e	D_i	t	l_o	h_o	h_o/t	s	l_t	F	σ_{II}	σ_{III}	s	l_t	F	σ_{II}	σ_{III}	s	l_t	F	σ_{II}	σ_{III}	s	l_t	F	σ_{II}	σ_{III}	s	F	σ_{OM}
C	22.5	11.2	0.60	1.40	0.80	1.33	0.12	1.28	160	-23	302	0.20	1.20	240	-14	488	0.40	1.00	370	98	897	0.60	0.80	426	336	1,227	0.80	444	-1,178
B	22.5	11.2	0.80	1.45	0.65	0.81	0.10	1.35	195	93	253	0.16	1.29	306	171	412	0.33	1.13	533	425	771	0.49	0.96	707	762	1,079	0.65	855	-1,276
A	22.5	11.2	1.25	1.75	0.50	0.40	0.08	1.68	424	224	234	0.13	1.63	693	383	384	0.25	1.50	1,330	815	737	0.38	1.37	1,929	1,296	1,059	0.50	2,509	-1,534
	23.0	8.2	0.70	1.50	0.80	1.14	0.12	1.38	[illegible]	37	245	0.20	1.30	279	87	397	0.40	1.10	448	295	733	[illegible]	0.90	[illegible]	626	1,007	0.80	602	-1,173
	23.0	8.2	0.80	1.55	0.75	0.94	0.11	1.44	214	92	237	0.19	1.36	332	175	384	0.38	1.18	560	457	714	0.56	0.99	719	846	991	0.75	842	-1,257
	23.0	8.2	0.90	1.70	0.80	0.89	0.12	1.58	311	125	277	0.20	1.50	486	233	449	0.40	1.30	829	589	837	0.60	1.10	1,078	1,066	1,164	0.80	1,279	-1,508

Some observations

- Coil compression spring vs disc spring





$$s1 = 0.15ho = 0.11\ mm \ \rightarrow Fa1 = 214\ N$$

$$s2 = 0.75ho = 0.56\ mm \ \rightarrow Fa2 = 719\ N$$

- Compression springs have a large deflection and spring rate range
- Disc springs require a large force to exert a small deflection

Lf = 88 mm, Fa = 35 N, La = 60 mm & Fo = 50 N, Lo = 50 N

Summary

- Springs of various types can be used in different engineering applications
- Spring rate, free or solid length, maximum length (for tension springs) or solid length (for compression springs) are key parameters to select a suitable spring with defined dimensions
- Coil springs (compression or tension) provide a wide range of spring rate, free length, maximum compression or extension for different operational conditions
- Disc springs (Belleville washers) can be a useful solution in situation which requires a large spring rate and relatively small deflection.
- The worked examples in selecting the **compression spring and disc spring** can be used in **your Individual (Gearbox Actuator) Project**.

References and resources

- R.G. Budynas and J.K. Nisbett, Shigley's Mechanical Engineering Design, 10th ed., 2015, Ch 10 (TJ230 BUD)
- Childs, P.R.N., Mechanical Design Engineering Handbook, 2nd ed., 2019, Ch15 (TJ230 CHI) (electronic edition also available on NUSearch)

- https://www.leespring.co.uk/
- **https://www.spirol.com/library/main_catalogs/SPIROL-Disc-Springs-us.pdf**
- https://www.schnorr.com/literature
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